

Performance Based Learning and Assessment Task

Georgia Peaches and California Dreamin'

I. ASSESSMENT TASK OVERVIEW & PURPOSE:

The students will use a geometric model and right angle trigonometry to solve a real life problem in several contexts. They will then use a PowerPoint presentation to set up their own real life problem and to present their problem and solution to a small group of fellow students. Students will learn to organize and consolidate their ideas and then to present these ideas. They will set up models to help them understand real life problems.

II. UNIT AUTHOR:

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III. COURSE:

Geometry

IV. CONTENT STRAND:

Geometry

V. OBJECTIVES:

SOL

G.8 The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

NCTM

- a) Use geometric modeling to solve problems
- b) Organize and consolidate their mathematical thinking through communication
- c) Analyze properties and determine attributes of two-dimensional objects
- d) Use technology

VI. REFERENCE/RESOURCE MATERIALS:

- a) Student Worksheet
- b) Listed Websites
- c) A computer and calculator

VII. PRIMARY ASSESSMENT STRATEGIES:

The task includes an assessment that can be used to guide student work. The teacher will use an adapted form of the assessment as a rubric to evaluate student work. Please find this assessment and rubric attached.

VIII. EVALUATION CRITERIA:

Please see attached assessment and rubric. A benchmark of exemplary work is also attached.

IX. INSTRUCTIONAL TIME:

The activity will take one 90 minutes for most students.

Georgia Peaches and California Dreamin'

Strand

Geometry

Mathematical Objective(s)

The students will use right triangle trigonometry to solve a real life problem.

Related SOL

- G.8

NCTM Standards

- Use geometric modeling to solve problems
- Organize and consolidate their mathematical thinking through communication
- Analyze properties and determine attributes of two-dimensional objects
- Use technology

Additional Objectives for Student Learning:

- ES 1d will be introduced or reinforced depending upon assessment obtained through Think, Pair, Share Activity.
- This activity enables the student to learn about the personal preferences of their classmates and so serves as a group builder and contributes to the collaborative atmosphere of the classroom.

Materials/Resources

- Classroom set of graphing calculators
- One computer with PowerPoint software and internet access for each pair of students
- One printed copy of the Student Worksheet for each student
- One printed copy of the assessment sheet and the rubric for each group
- LCD projector for the video clip to be shown during the whole group presentation time
- The small group activity on pages 5 to 8 should be copied into a new document and be posted on the class website for student to download.

Assumption of Prior Knowledge

This lesson assumes that the student has completed textbook study of right triangle trigonometry and is operating on Analysis level on Van Hiele scale with respect to the concepts of right triangles and their properties. Students should use the following vocabulary: longitude, latitude, right angle, hypotenuse, opposite adjacent. Students may find difficult or to understand how the sun's rays, the house and the shadow of the house make up right triangle.

Introduction: Setting Up the Mathematical Task

In this task, the students will apply their knowledge of right triangles to real life situations.

The students will spend about 15 minutes on a think, pair share activity, 15 minutes on a whole class lesson, 45 minutes completing the performance based assessment task and 10 minutes evaluating themselves and one another. The final 5 minutes will be devoted to a closure activity.

Each pair of students will design a PowerPoint presentation which requires that they understand the impact of latitude on the angle of the sun's rays. They will apply this knowledge and their knowledge of right triangle trigonometry as they plan their orchard. Additionally, they will model right triangle trigonometry in GeoGebra. Students will use the attached worksheet and teacher instructions to guide them in a think, pair, share activity which will enable them to draw on prior knowledge and to make their mathematical thinking public.

Student Exploration

Whole Class Sharing/Discussion

Earth and Sun Geometry

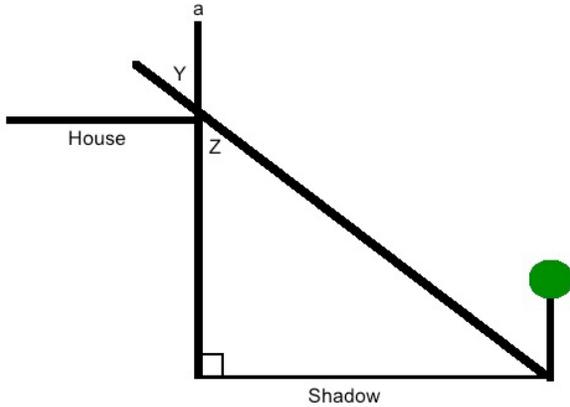
The class will watch the first five minutes of "Earth and Sun Geometry" and then be given the opportunity to change any of their answers on the think, pair, share worksheet. The class will then spend 5 minutes discussing their changes and the rationale for these changes.

<https://www.youtube.com/watch?v=9XH0VF2G0j8>

The teacher uses the following information to guide student exploration. Student use their worksheet to respond:

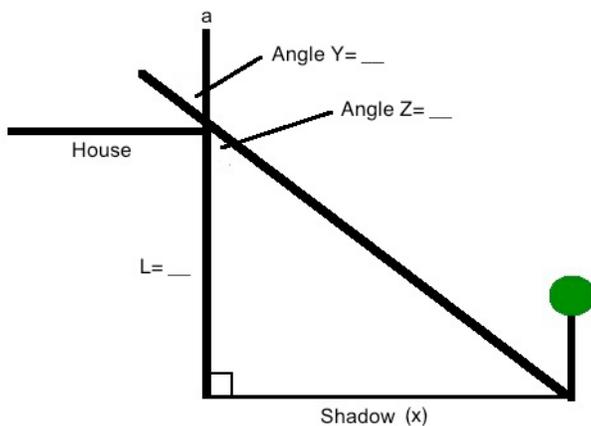
Uncle Bobby is a Georgia peach tree farmer by trade, and it is time to plant a new orchard. Bobby is a smart farmer, so he wants to maximize the amount of sun that his trees will receive. The students will help Bobby calculate the best way to plant his trees by answering the following questions.

1. Bobby wants to plant his trees as close to his house as possible, so that he can easily pick the ripe peaches. However, his one-story farmhouse is 15 feet tall, so he needs to make sure the trees are not in the shadow cast by the farmhouse. Help Bobby figure out how close to the farmhouse he can plant the trees, while ensuring that they remain in full sun at noon throughout the year. (Remember that the longest shadows occur on the winter solstice)



- a. In the diagram above, assume line a is a straight line. Are angles Y and Z equal? Explain.

- b. In order to calculate the length of the shadow, you must first calculate the value of angle Y (and Z). Angle Y is also known as the solar zenith angle, the angle the sun's rays make with respect to a vertical line. To calculate the solar zenith angle:
 - i. Find the latitude of Farmer Bobby (Georgia is about 33 N)
 - ii. Determine the latitude for which the sun is directly overhead at noon on the winter solstice (-23.5 N)
 - iii. Subtract the latitude where the sun is directly overhead from Farmer Bobby's latitude. This is the value for the solar zenith.
- c. Label the diagram below with the correct angles and side lengths



- d. Next, calculate the length of the shadow cast by the farmhouse at noon on the winter solstice using right triangle trigonometry. This is the minimum distance the trees can be planted from the farmhouse and still remain in full sun!

$$\tan(Z) = \frac{x}{L}$$

$$\tan(56.5) = \frac{x}{15}$$

$$x = (15)(1.511)$$

$$x = 22.66 \text{ ft}$$

Therefore, Farmer Bobby should plant his peaches about 23 ft away from his farmhouse.

Small Group Work

The teacher will introduce Small Group Work time by sharing the attached benchmark with the class. Students will then download the attached online instructions for next portion of the lesson from the class website. They will work in groups of two or three.

Student/Teacher Actions:

During this small group time students will be doing research on their computers and then designing their PowerPoints. The teacher will circulate around the room asking questions and responding to questions. It may be helpful to ask the students to sketch the appropriate triangles or to use a model of the earth and sun with a flashlight.

Monitoring Student Responses

Students will communicate their thinking and their new knowledge through their PowerPoint presentation. As they work on this presentation the teacher will assist students who have difficulties. In addition the teacher will ask the students who are ready to move forward to use the internet to find answers to the think, pair, share questions in previous sections. These students will receive extra credit for their work and may present it to the group if there is time.

Summary Activity

Each student groups will share the length of the shadow that they obtained. These will be listed on the board. The students will then make summary statements about how latitude impacts the angle of the sun's rays and thus the tangent values of their triangle.

Assessment List and Benchmarks

Please see attached assessment grid, rubric and benchmark.

Name _____

Date _____

Section _____

Student Worksheet for Georgia Peaches and California Dreamin'

Think, pair, share

You have 5 minutes to work with your partner and answer as many of the questions below as possible. After 5 minutes each group will report their observations.

1. Is there a relationship between shadow length and time of day?
2. Is there a relationship between shadow length and time of the year, e.g. the season?
3. How would your shadow change if you were standing on the equator?
4. How would your shadow change if you were standing on the north-pole?

You may change your answers above after the class discussion or the video if you would like.

Whole Class Activity

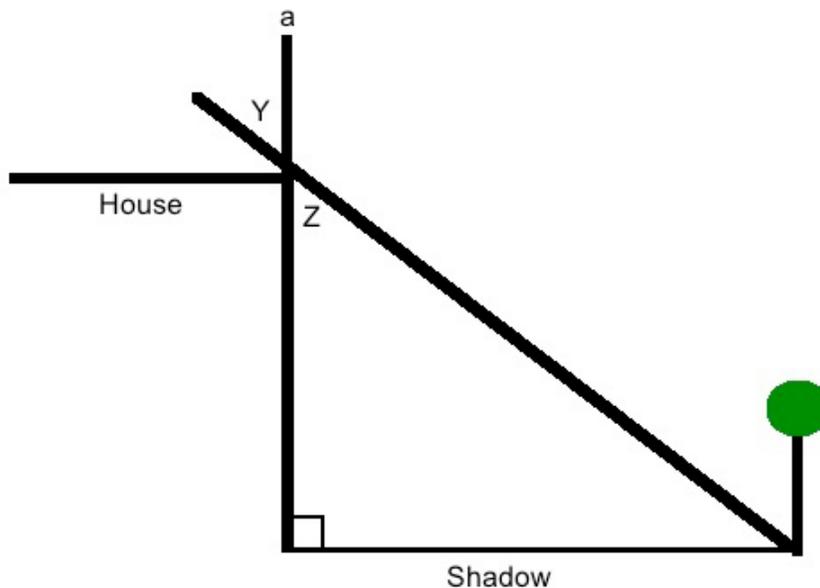
| | |
|--|---|
| | <ol style="list-style-type: none">1. Find the latitude of Farmer Bobby in GA2. Determine the latitude for which the sun is directly overhead at noon on the winter solstice3. Subtract the latitude where the sun is directly overhead from Farmer Bobby's latitude. This is the value for the solar zenith.4. Label the model to the right. |
|--|---|

- | | |
|--|---|
| | <p>5. Next, calculate the length of the shadow cast by the farmhouse at noon on the winter solstice using right triangle trigonometry. This is the minimum distance the trees can be planted from the farmhouse and still remain in full sun!</p> |
|--|---|

Answer Sheet for Whole Class Activity

Bobby is a Georgia peach tree farmer by trade, and it is time to plant a new orchard. Bobby is a smart farmer, so he wants to maximize the amount of sun that his trees will receive. The students will help Bobby calculate the best way to plant his trees by answering the following questions.

2. Bobby wants to plant his trees as close to his house as possible, so that it is easier to pick the ripe peaches. However, his one-story farmhouse is 15 ft tall, so he needs to make sure the trees are not in the shadow cast by the farmhouse. Help Bobby figure out how close to the farmhouse he can plant the trees, while ensuring that they remain in full sun at noon throughout the year. (Remember that the longest shadows occur on the winter solstice)



- e. In the diagram above, assume line a is a straight line. Are angles Y and Z equal?

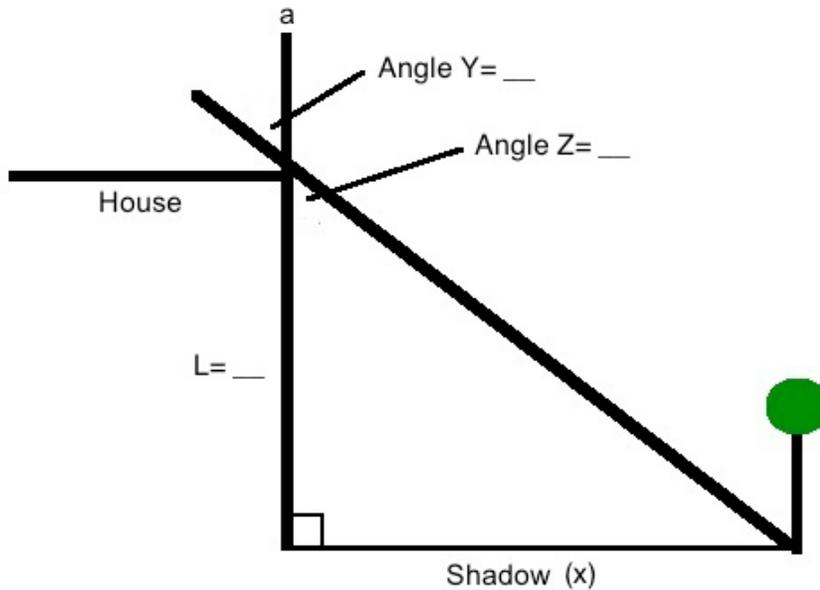
Yes

- f. In order to calculate the length of the shadow, you must first calculate the value of angle Y (and Z). Angle Y is also known as the solar zenith angle, the angle the sun's rays make with respect to the vertical. To calculate the solar zenith angle:
- iv. Find the latitude of Farmer Bobby (Georgia is about 33 N)
 - v. Determine the latitude for which the sun is directly overhead at noon on the winter solstice (-23.5 N)
 - vi. Subtract the latitude where the sun is directly overhead from Farmer Bobby's latitude.

$$33 \text{ N} - (-23.5 \text{ N}) = 56.5$$

Solar Zenith Angle $Y = 56.5$

- g. Label the diagram below with the correct angles and side lengths



$$\text{Angle } Y = 56.5^\circ$$

$$\text{Angle } Z = 56.5^\circ$$

$$L = 15 \text{ ft}$$

- h. Next, calculate the length of the shadow cast by the farmhouse at noon on the winter solstice using right triangle trigonometry. This is the minimum distance the trees can be planted from the farmhouse and still remain in full sun!

$$\tan(Z) = \frac{x}{L}$$

$$\tan(56.5) = \frac{x}{15}$$

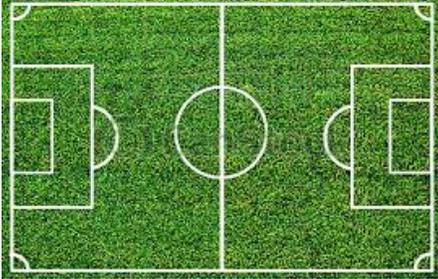
$$x = (15)(1.511)$$

$$x = 22.66 \text{ ft}$$

Therefore, Farmer Bobby should plant his peaches about 23 ft away from his farmhouse.

Small Group Instructions

While visiting Farmer Bobby you fell in love the idea of raising your own fruit. Farmer Bobby has agreed to help you establish an orchard in your backyard. You already have a vegetable garden at the very back of your lot. You want to maintain most of the yard as is because you and your friend love to play one-on-one soccer there. This means you want to put in a line of trees as close as possible to the vegetable garden without shading the garden. Assume the following configuration of your yard and use the questions below to begin plan your dream location and orchard:

| | |
|-----------------------|--|
| South | |
| Your House |  |
| Backyard Soccer Field |  <p data-bbox="607 1129 831 1146">© Can Stock Photo - csp10043722</p> |
| Line of Trees |  |
| Vegetable Garden |  <p data-bbox="509 1755 683 1787">Downloaded from Dreamstime.com</p> <p data-bbox="959 1755 1057 1787">©11986 8081241 Dreamstime.com</p> |
| North | |

Use the following worksheet to gather the information you will need to for your PowerPoint presentation.

1. Ready for a change of pace? Use google maps to consider a move and then pick your dream location. Where do you want to live?

<https://www.google.com/maps/@38.0033855,-79.4209249,7z?hl=en>

2. Using the zip code data base, find your new zip code. What is your new zip code?

<http://www.unitedstateszipcodes.org/>

3. Use Stark Bro's website or another tree supplier website to type in your zip code and find out what trees will grow in your new hardiness zone. Pick a dream tree. How tall it is expected to get?

<http://www.starkbros.com/products/fruit-trees>

4. Determine the latitude of your dream house. What is the latitude? Use the formula below to determine the angle of the sun.

$$\text{your latitude} - -23.5 = \text{the angle of the sun}$$

What is the angle of the sun's rays on winter solstice at your dream location?

5. Next use the following formula to determine how far your row of trees should be from your vegetable garden

$$\tan(a) = \frac{s}{h}$$

a = angle of the sun; s = shadow length; h = height of your dream tree

Use your answers above to guide your creation of a PowerPoint with the following components:

- Slide #1: Give the name of your tree and your own name on your title slide.
- Slide #2: Tell why you chose your fruit and include a picture of the fruit.
- Slide #3: Give the location of your orchard, tell why you chose this location and include a picture of your location.
- Slide #4: Give your latitude and computer the distance the line of trees should be from the garden

- Slide #5: Use GeoGebra to draw the triangle produced by the tree and its shadow and check to see if the angle produced is the Solar Zenith angle matches the angle you had calculated.

You have 45 minutes to complete this assignment. As you are working complete the assessment sheet that your teacher has given you. You will be told when to present your PowerPoint to the group you have been assigned. They will fill out the rubric and share it with you. Next they will present to you and you will fill out the rubric. Your project is due tomorrow so that you may edit your presentation based on the feedback you receive if they would like to do so.

The Georgia Peaches Assessment List

| Num. | Element | Point Value | Earned Assessment | |
|------|--|-------------|-------------------|---------|
| | | | Self | Teacher |
| 1 | The fruit or nut tree chosen matched the hardiness zone to the city chosen | 2 | | |
| 2 | The student correctly identified the Latitude of their chosen city | 2 | | |
| 3 | Mathematical formulas and calculations are correct | 2 | | |
| 4 | Work is neat and attractive. | 2 | | |
| 5 | The PowerPoint is complete | 2 | | |
| 6 | The PowerPoint clearly conveys the needed information | 2 | | |

Rubric for Georgia Peaches and California Dreaming

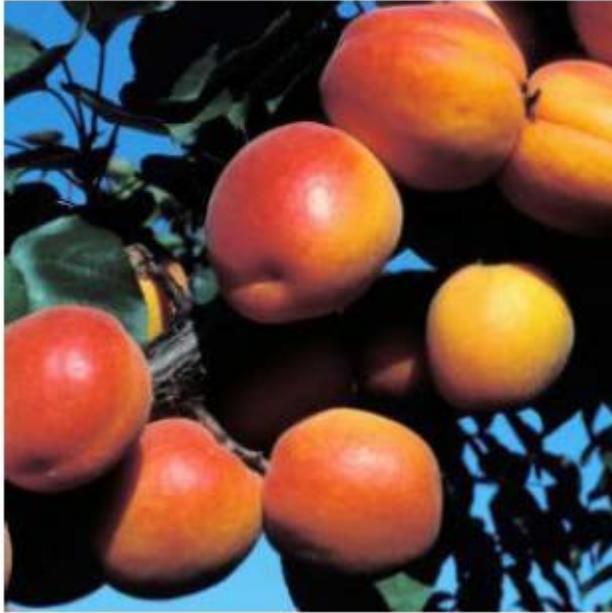
| # | Element | 0 | 1 | 2 |
|---|---|---|--|--|
| 1 | The fruit or nut tree matched the hardiness zone of the city chosen | The fruit or nut tree is far from the correct hardiness zone of the city chosen | The fruit or nut tree is near the hardiness zone to the city chosen | The fruit or nut tree chosen matched the hardiness zone to the city chosen |
| 2 | The student correctly identified the Latitude of their chosen city | The student did not identify the Latitude of their chosen city | The student was close to the correctly Latitude of their chosen city | The student correctly identified the Latitude of their chosen city |
| 3 | Mathematical formulas and calculations are correct | None of the Mathematical formulas and calculations are correct | Some of the Mathematical formulas and calculations are correct | All of the Mathematical formulas and calculations are correct |
| 4 | Work is neat and attractive. | None of the Work is neat and attractive. | Some of the Work is neat and attractive. | All of the Work is neat and attractive. |
| 5 | The PowerPoint is complete | Many parts are missing from the PowerPoint | Many parts are missing from the PowerPoint | The PowerPoint is complete |
| 6 | The PowerPoint clearly conveys the needed information | The PowerPoint does not convey the needed information | The PowerPoint conveys the needed information | The PowerPoint clearly conveys the needed information |

Benchmark

Please note: Each page below is representative of one PowerPoint slide in the Benchmark.

My Robada Apricot Orchard

I am choosing apricots because I love the flavor and they are good for canning.



My orchard will be in Denver because I have always wanted to live near the Rockies.



Denver Latitude 39.7

$$39.7 N - (-23.5 N) = 63.2$$

Solar Zenith Angle $a = 63.2$

$$\tan(a) = \frac{s}{h}$$

$a =$ angle of the sun; $s =$ shadow length; $h =$ height of your dream tree

$$\tan(63.2) = \frac{s}{15}$$

$$s = 15(1.97966)$$

$$s = 29.69$$

Therefore, I should plant my apricot trees in a line that is 30 feet away from my vegetable garden.

Double Checking Work with a GeoGebra Illustration

